Exercises 16 B, P405

Exercises here depend on $a = v\frac{dv}{ds}$ as well as $a = \frac{dv}{dt}$ & $v = \frac{ds}{dt}$, depending on what info we are given.

(1)

(2) $V = 2S - 3$, $a = V\frac{dv}{ds} = V(2)$

when $S = 4$, $V = 5$ so $a = (5)(2) = 10\,m/s^2$

(3) $V = \frac{S^2}{15}$ & $S = 3$ when $t = 0$.

So $V = \frac{ds}{dt} = \frac{S^2}{15} \Rightarrow \int \frac{15}{S^2}\,ds = \int dt$

$\Rightarrow -\frac{15}{S} + C = t$

By condition given: $-\frac{15}{3} + C = 0 \Rightarrow C = 5$

$\therefore t = -\frac{15}{S} + 5$. So when $S = 10$, $t = 3\frac{1}{2}$ sec

(4)

(5) $V = S - 2$, when $a = 3$ find S.

Now, $a = V.\frac{dv}{ds} \Rightarrow a = (S-2)(1)$. So $3 = S - 2$

$\Rightarrow S = 5\,m$

(6) $V = 2s + 3$ & $s = 0$ when $t = 0$

So $v = \frac{ds}{dt} \Rightarrow 2s + 3 = \frac{ds}{dt} \therefore \int dt = \int \frac{1}{2s+3}\, ds$

So $t = \frac{1}{2}\ln|2s+3| + c$. But $s = 0$ when $t = 0$ so,

$0 = \frac{1}{2}\ln|3| + c \Rightarrow c = -\frac{1}{2}\ln 3.$

$\therefore t = \frac{1}{2}\ln|2s+3| - \frac{1}{2}\ln 3$

$\therefore$ when $s = 3$, $t = \frac{1}{2}\ln 9 - \frac{1}{2}\ln 3 = \ln 3 - \frac{1}{2}\ln 3 = \frac{1}{2}\ln 3$ m

(7) ✓

(8) $a = \frac{40}{s^2}$ & $V = 0$ when $s = 10$

Now, $a = v \cdot \frac{dv}{ds} \Rightarrow \frac{40}{s^2} = v \cdot \frac{dv}{ds}$

So $\int \frac{40}{s^2}\, ds = \int v\, dv \Rightarrow -\frac{40}{s} + c = \frac{v^2}{2}$

By $v = 0$ when $s = 10$: $c = 4$, so

$$\frac{v^2}{2} = -\frac{40}{s} + 4$$

So when $s = 20$, $v = 2$ m/s

(9) ✓

(10) $a = \dfrac{1}{s+2}$ & $v=0$ when $s=0$

So $a = v.\dfrac{dv}{ds} \Rightarrow \dfrac{1}{s+2} = v.\dfrac{dv}{ds}$

$\therefore \displaystyle\int \frac{1}{s+2}\,ds = \int v.dv \Rightarrow \ln(s+2) + c = \frac{v^2}{2}$

by $v=0$ when $s=0$: $c = -\ln 2$ so,

$$\frac{v^2}{2} = \ln(s+2) - \ln 2$$

When $v = 1.5$ m/s : $\dfrac{2.25}{2} = \ln\left(\dfrac{s+2}{2}\right) \Rightarrow e^{1.125} = \dfrac{s+2}{2}$

$\therefore s = 4.16$m

(11) $a = s+2$ & $v=2$ when $s=0$.

(a) $a = v.\dfrac{dv}{ds} \Rightarrow s+2 = v.\dfrac{dv}{ds}$, $\therefore \displaystyle\int s+2\,ds = \int v.dv$

$\therefore \dfrac{s^2}{2} + 2s + c = \dfrac{v^2}{2}$; but $s=0$ & $v=2$, $\therefore c=2$,

hence $\dfrac{v^2}{2} = \dfrac{s^2}{2} + 2s + 2 \Rightarrow v^2 = s^2 + 4s + 4 = (s+2)^2$

$\therefore v = (s+2)$ m/s

(b) $v = \dfrac{ds}{dt}$, so $s+2 = \dfrac{ds}{dt} \Rightarrow \displaystyle\int dt = \int \frac{1}{s+2}\,ds$.

$\therefore$ $t = \ln(s+2) + c$; But $t=0$ when $s=0$,
($t=0$ not stated but can be assumed when $s=0$),
so $c = -\ln 2$ & $\therefore t = \ln(s+2) - \ln 2 \Rightarrow t = \ln\left(\dfrac{s+2}{2}\right)$.

(12) $a = 4s + 2$ with $v = 1$ when $s = 0$.

(a) $a = v \cdot \frac{dv}{ds} = 4s + 2$, $\therefore \int v \, dv = \int 4s + 2 \, ds$

$\Rightarrow \frac{v^2}{2} = 2s^2 + 2s + c$. But $v = 1$ when $s = 0$, so

$c = \frac{1}{2}$, and $\therefore \frac{v^2}{2} = 2s^2 + 2s + \frac{1}{2}$

$\Rightarrow v^2 = 4s^2 + 4s + 1 = (2s+1)^2$

So $v = 2s + 1$. When $s = 3$, $v = 7$ m/s

(b) $v = \frac{ds}{dt} = 2s + 1$, $\therefore \int \frac{1}{2s+1} ds = \int dt$

$\therefore \frac{1}{2} \ln (2s+1) + c = t$

But $t = 0$ when $s = 0$, so $c = 0$, Hence

$t = \frac{1}{2} \ln (2s+1)$

& when $s = 3$, $t = \frac{1}{2} \ln 7$ sec.

(13) /

(14) $a = \frac{7}{v}$ & $v = 3$ when $s = 0$.

So $a = v \cdot \frac{dv}{ds} = \frac{7}{v} \Rightarrow \int v^2 dv = \int 7 \, ds$

But $v=3$ when $s=0$, so $c=9$, hence

$$\frac{1}{3}v^3 = 7s+9 \Rightarrow v^3 = 21s+27$$

Now, $v=6$ so $s=9m$.

(15) $a = \frac{2}{3v^2}$ & $v=0$ when $t=0$.

So $a = \frac{dv}{dt} = \frac{2}{3v^2} \Rightarrow \int 3v^2 dv = \int 2\,dt$

$\therefore v^3 = 2t + c$. But $v=0$ when $t=0 \Rightarrow c=0$.

So $v^3 = 2t$. When $v=4$, $t=32$ sec.

(16) Same as (15). Relevant Equation is $t = \ln(2+v) - \ln 2$

So when $v=12$, $t = \ln 7$.

(17) $a = \frac{v^2}{5}$ & $v=1$ when $s=0$

So $a = v\frac{dv}{ds} = \frac{v^2}{5} \Rightarrow \int \frac{1}{v}dv = \frac{1}{5}\int ds$

$\therefore \ln v = \frac{1}{5}s + c$. But $v=1$ when $s=0 \Rightarrow c=0$.

$\therefore \ln v = \frac{1}{5}s$. When $v=6$, $s = 5\ln 6$ m.

(18) $a = 5 + 2v$ & $v=0$ when $s=0$

So $a = v\dfrac{dv}{ds} = 5+2v \Rightarrow \displaystyle\int \frac{v}{5+2v}\,dv = \int ds$.

use method of substitution:

Let $u = 5+2v$. Then $v = \dfrac{u-5}{2}$. Hence we have

$\displaystyle\int \frac{u-5}{2} \cdot \frac{1}{u} \cdot \frac{1}{2}\,du = \int ds$, where $\frac{1}{2}du$

comes from differentiating $u = 5+2v$.

$\therefore \displaystyle\int \frac{1}{4}\,\frac{u-5}{u}\,du = \int ds \Rightarrow \int \frac{1}{4} - \frac{5}{4u}\,du = \int ds$

$\therefore \frac{1}{4}u - \frac{5}{4}\ln u = s + c$. But $u = 5+2v$ so

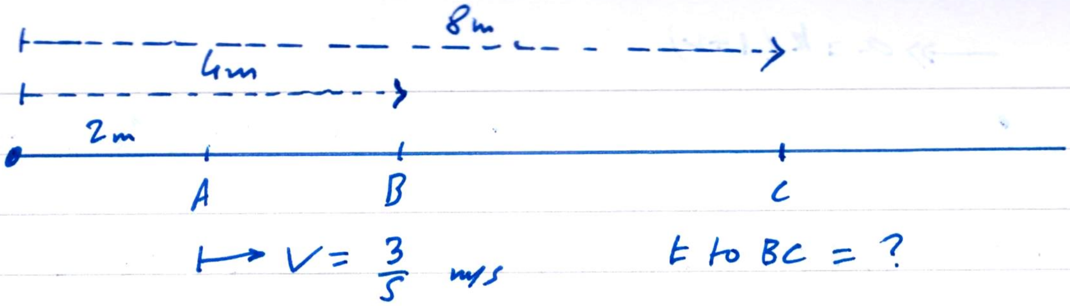
So $\frac{1}{4}(5+2v) - \frac{5}{4}\ln(5+2v) = s + c$. But $s = v = 0$

So $c = \frac{5}{4} - \frac{5}{4}\ln 5$. Hence

$$\frac{1}{4}(5+2v) - \frac{5}{4}\ln(5+2v) = s + \frac{5}{4} - \frac{5}{4}\ln 5$$

when $v=5$, $s = 1.127$ m.

(19)



given : From A : $v = \frac{3}{s}$ m/s & $v = 0$ when $s = 2$

So $v = \frac{ds}{dt} = \frac{3}{s} \Rightarrow \int s\, ds = 3 \int dt$

$\therefore \frac{s^2}{2} = 3t + c$. But $v = 0, s = 2$ so $c = 2$

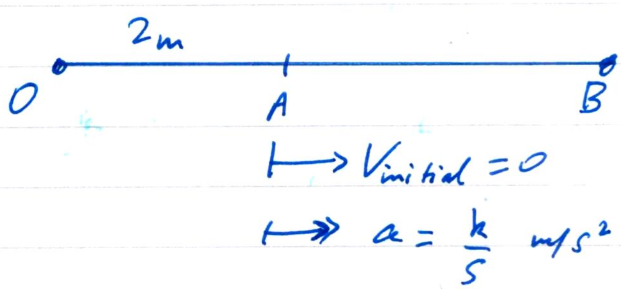
hence $\frac{s^2}{2} = 3t + 2 \Rightarrow s^2 = 6t + 4$

$O \to B$: $s = 4$, so $4^2 = 6t + 4 \Rightarrow t = 2$ sec

$O \to C$: $s = 8$, so $8^2 = 6t + 4 \Rightarrow t = 10$ sec

So time from $B \to C$ = time $O \to C$ − time $O \to B$
$= 10 - 2 = 8$ sec

(20)



given : $v = 0$ when $s = 2$. So $a = v.\frac{dv}{ds} = \frac{k}{s}$

$\therefore \int v\, dv = k \int \frac{1}{s}\, ds \Rightarrow \frac{v^2}{2} = k.\ln s + c$

But $v = 0$ when $s = 2 \Rightarrow c = -k \ln 2$. Hence

$\frac{v^2}{2} = k \ln s - k \ln 2 \Rightarrow v^2 = 2k \ln \frac{s}{2}$ ✓

$\longrightarrow\!\!\!\!\rightarrow a = k(1+v)$

(21)

& $v=0$ when $s=0$

Now $a = v.\dfrac{dv}{ds} = k(1+v)$, $\therefore \displaystyle\int \frac{v}{1+v}\,dv = k\int ds$

Let $u = 1+v$ (use of method of substitution)

So $du = dv$

& $u-1 = v$

$\therefore$ we have $\displaystyle\int \frac{u-1}{u}\,du = k\int ds$

i.e. $\displaystyle\int 1 - \frac{1}{u}\,du = ks + c$

i.e. $u - \ln u = ks + c$

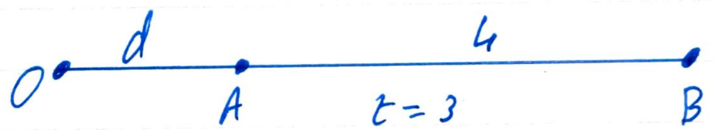
$\therefore (1+v) - \ln(1+v) = ks + c$

But $v=0$ when $s=0$ so $c = 1$. Hence

$(1+v) - \ln(1+v) = ks + 1$

$\Rightarrow \quad s = \dfrac{1}{k}\left[v - \ln(1+v)\right]$ ✓

(22)



$\mapsto v = \dfrac{8}{3+s}$ and $s=0,\ t=0$.

Now $v = \dfrac{ds}{dt} = \dfrac{8}{3+s} \Rightarrow \displaystyle\int 3+s\,ds = \int 8\,dt$

$\therefore 3s + \dfrac{s^2}{2} = 8t + c$

But $s=0$ at $t=0$ so $c=0$, $\therefore\ 3s+\frac{s^2}{2}=8t$

hence $O\to A$: $3d+\frac{d^2}{2}=8t$ ①

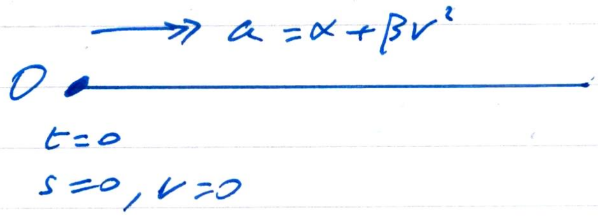
& $O\to B$: $3(d+4)+\frac{(d+4)^2}{2}=8(t+3)$

i.e. $3d+12+\frac{1}{2}(d^2+8d+16)=8t+24$ ②

By ①, ② simplifies to $3d+12+\frac{1}{2}d^2+4d+8=3d+\frac{d^2}{2}+24$

$\therefore\quad 12+4d+8=24 \quad\Rightarrow\quad d=1\text{m}$

23



Now, $a=v.\frac{dv}{ds}=\alpha+\beta v^2$

$\therefore\ \int\frac{v}{\alpha+\beta v^2}\,dv=\int ds \quad\Rightarrow\quad \frac{1}{2\beta}\ln(\alpha+\beta v^2)=s+c$

But $s=0$ & $v=0$, so $\frac{1}{2\beta}\ln(\alpha+\beta v^2)=s+\frac{1}{2\beta}\ln\alpha$.

Simplify to isolate v^2: $\ln(\alpha+\beta v^2)=2\beta s+\ln\alpha$

$\therefore\ \alpha+\beta v^2=e^{2\beta s+\ln\alpha}=\alpha e^{2\beta s}$

hence $v^2=\frac{1}{\beta}(\alpha e^{2\beta s}-\alpha)=\frac{\alpha}{\beta}(e^{2\beta\alpha}-1)$ ✓

(24)

$\longrightarrow a = 5 - 2v$ $v = 0$ when $t = 0$

O

Now $a = \dfrac{dv}{dt} = 5 - 2v$, $\therefore \displaystyle\int \frac{1}{5-2v}\,dv = \int dt$

hence $-\frac{1}{2}\ln(5-2v) = t + c$. But $v=0$ when $t=0$ so

$c = -\frac{1}{2}\ln 5$. Therefore

$$t - \frac{1}{2}\ln 5 = -\frac{1}{2}\ln(5-2v)$$

$$\Rightarrow \quad t = \frac{1}{2}\ln 5 - \frac{1}{2}\ln(5-2v) = \frac{1}{2}\ln\left(\frac{5}{5-2v}\right) \checkmark$$

From this: $\quad 2t = \ln\left(\dfrac{5}{5-2v}\right)$

$$\therefore \quad e^{2t} = \frac{5}{5-2v} \quad \Rightarrow \quad 5 - 2v = 5e^{-2t}$$

$$\therefore \quad 5 - 5e^{-2t} = 2v \quad \Rightarrow \quad v = \frac{5}{2}\left(1 - e^{-2t}\right)$$

Then as $t \to \infty$ $\quad v \to 5/2$ $\quad$ So max velocity $v = 5/2$ m/s